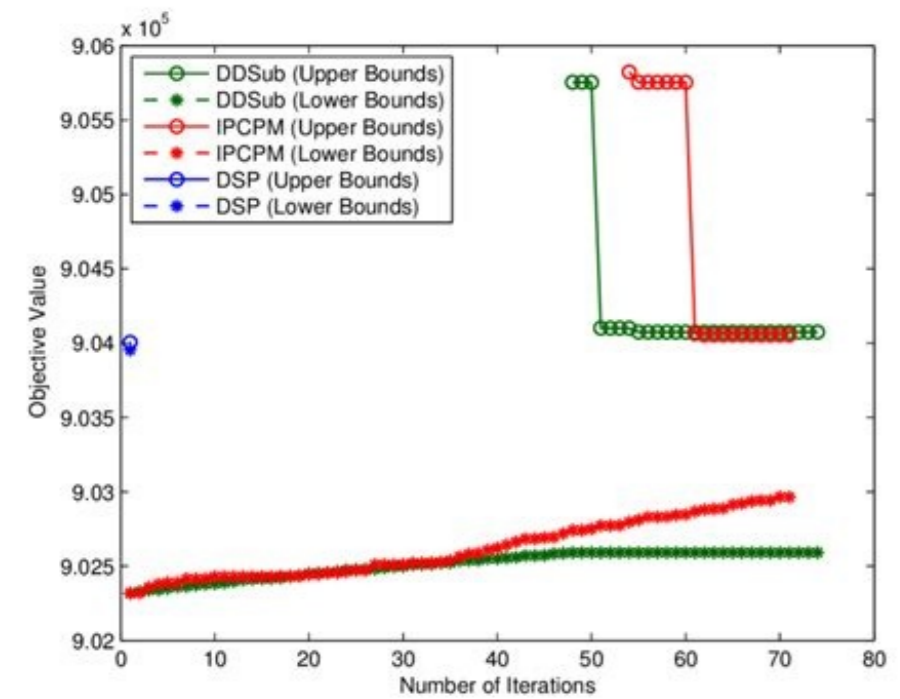
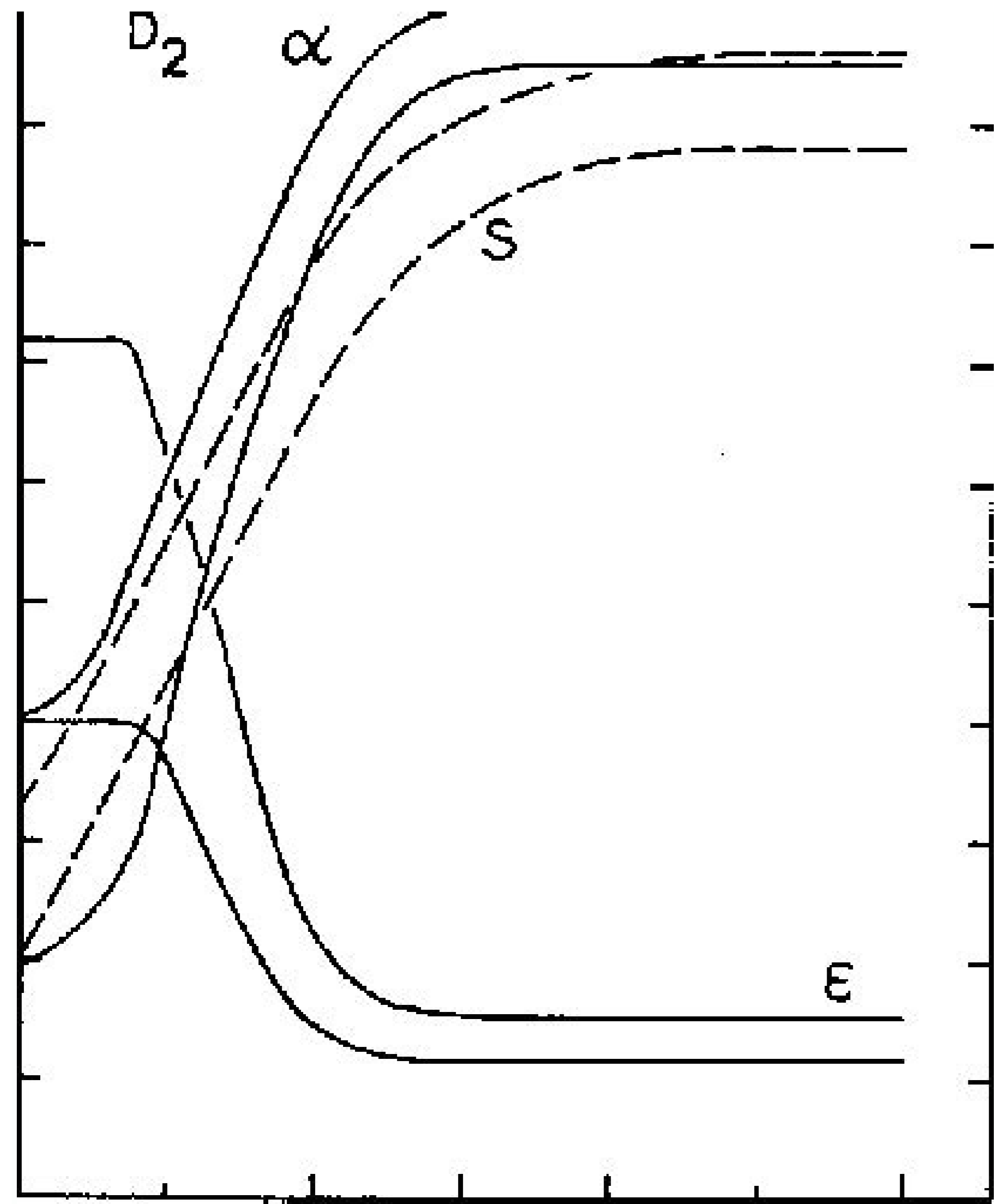


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Statistics of the MLE and Approximate Upper and Lower Bounds – Part 2: Threshold Computation and Optimal Signal Design

Achraf Mallat, Member, IEEE, Sinan Geziçi, Senior Member, IEEE, Davide Dardari, Senior Member, IEEE, and Luc Vandendorpe, Fellow, IEEE.

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Abstract—Threshold and ambiguity phenomena are studied in Part 1 of this work Mallat et al. where approximations for the mean-squared-error (MSE) of the maximum likelihood estimator are proposed using the method of interval estimation (MIE), and where approximate upper and lower bounds are derived. In this part we consider time-of-arrival estimation and we employ the MIE to derive closed-form expressions of the begin-ambiguity and asymptotic signal-to-noise ratio (SNR) thresholds with respect to some features of the transmitted signal. Both handset and pushdown pulses are considered. We prove that the begin-ambiguity threshold depends only on the shape of the envelope of the ACR, whereas the end-ambiguity and asymptotic thresholds only on the shape of the ACR. We exploit the results on the begin-ambiguity and asymptotic thresholds to optimize, with respect to the available SNR, the pulse that achieves the minimum achievable MSE. The results of this paper are valid for various estimation problems.

Index Terms—Nonlinear estimation, threshold and ambiguity phenomena, maximum likelihood estimator, mean-squared-error, signal-to-noise ratio, time-of-arrival, optimal signal design.

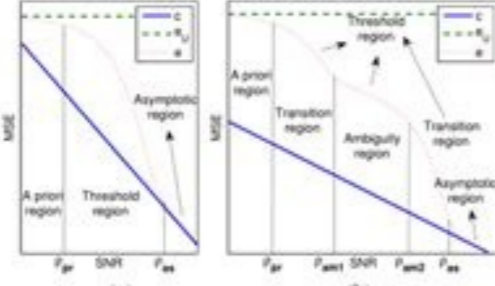


Fig. 1. SNR regions (a) α priori, threshold and asymptotic regions for non-oscillating ACR. (b) α priori, ambiguity and asymptotic regions for oscillating ACR. The MSE of the MLE estimator in the α priori region is the achievable MSE. $\rho_{p_1}, \rho_{p_2}, \rho_{p_3}, \rho_{p_4}$ at α priori, begin-ambiguity, end-ambiguity and asymptotic thresholds.

I. INTRODUCTION

NONLINEAR deterministic parameter estimation is subject to the threshold effect [Ziv and Zakai (1966), Zhou and Schultheis (1981), Weiss and Weinstein (1983), Weinstein and Weiss (1984), Ziv and Schultheis (1993), 1994], Sadler and Kozick (2006), Sadler et al. (2007)]. Due to this effect the signal-to-noise ratio (SNR) axis can be split into three regions as illustrated in Fig. 1(a):

- 1) α priori region: Region in which the estimator becomes uniformly distributed in the α priori domain.
- 2) Threshold region: Region of transition between the α priori and asymptotic regions.
- 3) Asymptotic region: Region in which an asymptotically efficient estimator, such as the maximum likelihood estimator (MLE), achieves the Cramer-Rao lower bound (CRLB). Otherwise, the estimator achieves its own asymptotic mean-squared-error (MSE) (e.g. MLE with

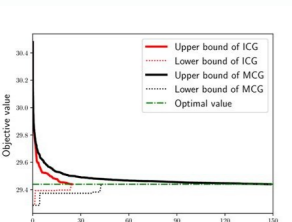
random signals and finite snapshots Renaux et al. (2006, 2007)].

When the autocorrelation (ACR) with respect to (w.r.t.) the unknown parameter is oscillating, five regions can be identified as shown in Fig. 1(b): 1) the α priori region, 2) the α priori ambiguity transition region, 3) the ambiguity region, 4) the ambiguity-asymptotic transition region, and 5) the asymptotic region. The MSE achieved in the ambiguity region is approximately equal to the envelope CRLB (ECRLB). In Figs. 1(a) and 1(b), $\rho_{p_1}, \rho_{p_2}, \rho_{p_3}$ and ρ_{p_4} , respectively, denote the α priori, begin-ambiguity, end-ambiguity and asymptotic thresholds, determining the limits of the defined regions.

As the evaluation of the statistics of most estimators such as the MLE is often unattainable in the threshold region, many lower bounds have been proposed [Van Trees and Bell (2007), Renaux (2006)] for both deterministic (the unknown parameter has only one possible value) and Bayesian (the unknown parameter follows a given α priori distribution) estimation in order to be used as benchmarks and to describe the behavior of an estimator in that region.

Threshold computation is considered in Weiss and Weinstein (1983), Weinstein and Weiss (1984) where the α priori, begin-ambiguity, end-ambiguity and asymptotic thresholds are computed based on the Ziv-Zakai lower bound (ZZLB); the ZZLB evaluates accurately the asymptotic threshold and detects roughly the ambiguity

Achraf Mallat and Luc Vandendorpe are with the KTEAM Institute, Université Catholique de Louvain, Belgium. Email: {Achraf.Mallat, Luc.Vandendorpe}@uclouvain.be.
Sinan Geziçi is with the Department of Electrical and Electronics Engineering, Bilkent University, Ankara 06800, Turkey. Email: gezi@ee.bilkent.edu.tr.
Davide Dardari is with DEL-CNT at University of Bologna, Italy. Email: dardari@del.unibo.it.
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Descriptors	Lower bound	Upper bound
\sqrt{F}	0.1	0.3
N	100	300
F_d	5	30
e_i	1	3

I.J.B.F. Adan, J. Wessels and W.H.M. Zijm, Analysis of the shortest symmetrical queue problem, Stochastic Mod. 6 (1990) 691-713. Google Scholar I.J.B.F. Adan, J. Wessels and W.H.M. Zijm, Matrix-geometric analysis of the shortest queue problem with threshold jockeying, Oper. 13 (1993) 107-112. Article Google Scholar J.P.C. Blanc... The power series algorithm applied to the shortest model, Oper. Res. 40 (1992) 157-167. MathSciNet Article Google Scholar J.W. Cohen and O.J. Boxma, Boundary Value Problems in Queueing System Analysis (North-Holland, Amsterdam, 1983). Google Scholar B.W. Conolly, The problem of the highway tail, J. Appl. Prob. 21 (1984) 394-403. Google Scholar N.M. van Dijk and B.F. Lamond, Simple limits for exponential queues to single server, Oper. Res. 36 (1988) 470-477. Google Scholar N.M. van Dijk and J. van der Wal, simple limits and results of monotonicity for finite multi-server tandem code, Queueing Syst. 4 (1989) 1-16. Article Google Scholar G. Fayolle and R. Iasnogorodski, Two Processors coupled: the reduction of a problem Riemann-Hilbert, Z. Wahrsch. Verw. Gebiete (1979) 325-351. L. Flatto and H.P. McKean, Two parallel queues with equal maintenance rates, Science Report, RC5916, IBM (1977). L. Flatto and H.P. McKean, Two tails in parallel, Comm. Pure Appl. Math. 30 (1977) 255-263. Google Scholar G.J. Foschini and J. Salz, A basic dynamic routing problem, IEEE Trans. Commun. COM-26 (1978) 320-327. Article Google Scholar I. Gertsbakh, The shortest queue problem: A numerical study using the matrix-geometric solution, Eur. J. Oper. Res. 15 (1984) 374-381. Article Google Scholar W.K. Grassmann, Transient and constant status results for two parallel queues, OMEGA Int. J. Manag. Sci. 8 (1980) 105-112. Article Google Scholar W.K. Grassmann and Y. Zhao..., A numerically stable algorithm for two code server models, J. 9891 (11 murtceps ro, sboj fo rebnum eht ni krowten gniewueq laineopse desolec a fo tughpuorht eht fo yticinotonom, Law Res. Nav. 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In: probability in the spaces of Banach. Marmalade. Mathematics. SOC. 7. 455An A tow a e ce537 (1994) Crossref a Mathscinet a e Google Scholar a e, M., Yukich, J.: The integration of the cost of the exponential transport square. Ann. App. Probab. 3, 1100a e a .- a e ce1111 (1993) Crossref a e Google Scholar Yukich, J.: Some generalizations of the two-sampled Euclidean combination problem. In: probability in the spaces of Banach, vol. 8. Brunswick, Me, 1991. Program. Probab., Vol. 30, pages 55 - 66. Birkh bets, Boston (1992) Google Scholar Numpy and Scipy provide a complete means to work with 2D data. Pandas has the specific class data frame to manage 2D labeled data. Start creating an array number 2D: >>>>> a = np.array([(1, 1, 1), ..., [2, 3, 1], ..., [4, 9, 2], ..., [8, 27, 4], ..., 53.400000000001 As you can see, statistics are obtained (such as average, median or variance) on all data in array a. Sometimes, this behavior is what you want, but in some cases, you will want to calculate these quantities for each line or column of your 2D array. The functions and methods you have used so far have an optional parameter called Axis, which is essential to manage 2D data. The axis can take one of the following values: axis = nobody says to calculate the statistics above all data in the array. The above examples work this way. This is what this is about. Is often the default in NumPy, axis=0 says to calculate the statistics across all rows, that is, for each column of the array. This behavior is often the default for SciPy statistical functions. axis=1 says to calculate the statistics across all columns, that is, for each row of the array. LeteAAAs see axis=0 in action with np.mean(): >>>>> np.mean(a, axis=0) array([6.2, 8.2, 1.8]) >>> a.mean(axis=0) array([6.2, 8.2, 1.8]) The two statements above return new NumPy arrays with

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